ΑΣΚΗΣΗ Κ2

1. Να λυθούν τα προβλήματα συνοριακών τιμών:

A) $\Delta u(\rho,\varphi)=0, 1<\rho<2, 0\le\varphi<2\pi$

$u(1,\varphi)=0, 0\le\varphi<2\pi, u(2,\varphi)=1, 0\le\varphi<\pi, u(2,\varphi)=-1, \pi\le\varphi<2\pi.$

B) $\Delta u(\rho,\varphi)=0, 0\le\rho<2, 0\le\varphi<2\pi, \dfrac{\partial u(\alpha,\varphi)}{\partial\rho}=\mathrm{A}+\cos\varphi.$

C) $\Delta u(\rho,\varphi)=\rho^2\cos\varphi, 1\le\rho<2, 0\le\varphi<2\pi, \dfrac{\partial u(1,\varphi)}{\partial\rho}=A+2, \dfrac{\partial u(2,\varphi)}{\partial\rho}=0.$

2. Να λυθούν τα προβλήματα αρχικών-συνοριακών τιμών:

$a)\Delta u(x_1,\mathrm{x}_2,t)=\dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}, (x_1,\mathrm{x}_2)\in(0,1)\times(0,1), \mathrm{t}>0, u(x_1,\mathrm{x}_2,0)=\sin\pi x_1\sin\pi\mathrm{x}_2,$

$u_t(x_1,\mathrm{x}_2,0)=0, u(0,\mathrm{x}_2,t)=u(1,\mathrm{x}_2,t)=u(x_1,0,t)=u(x_1,1,t)=0$

$\mathrm{b})\Delta u(x,t)=\dfrac{\partial u}{\partial t}, x\in(-\infty,\infty), \mathrm{t}>0, u(x,0)=e^{-x^2},$

$u, u_x\to 0, |x|\to\infty.$

$\mathrm{c})\Delta u(x,\mathrm{y},t)=\dfrac{\partial u}{\partial t}, (x,y,t)\in(-\infty,\infty)\times(-\infty,\infty), \mathrm{t}>0, u(x,\mathrm{y},0)=2e^{-(x^2+\frac{y^2}{3})},$

$u, u_x, u_y\to 0, \mathrm{x}^2+y^2\to\infty.$

$\mathrm{d})\Delta u(x,t)=\dfrac{\partial u}{\partial t}, x\in(0,\infty), \mathrm{t}>0, u(x,0)=0, u_x(0,\mathrm{t})=\mathrm{cost}$

$u, u_x\to 0, \mathrm{x}\to\infty.$

$\mathrm{e})\Delta u(x,t)=\dfrac{\partial u}{\partial t}, x\in(0,\infty), \mathrm{t}>0, u(x,0)=0, u(0,\mathrm{t})=5\,\mathrm{cost}$

$u, u_x\to 0, \mathrm{x}\to\infty.$